

Time Variation of the Fine Structure Constant in the Spacetime of a Cosmic Domain Wall

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Abstract

The gravitational field produced by a domain wall acts as a medium with spacetime-dependent permittivity ε . Therefore, the fine structure constant $\alpha = e^2/4\pi\varepsilon$ will be a time-dependent function at fixed position. The most stringent constraint on the time-variation of α comes from the natural reactor Oklo and gives $|\dot{\alpha}/\alpha| < few \times 10^{-17} \text{ yr}^{-1}$. This limit constrains the tension of a cosmic domain wall to be less than $\sigma \lesssim 10^{-2} \text{ MeV}^3$, and then represents the most severe limit on the energy density of a cosmic wall stretching our Universe.

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1. Introduction

The physics of topological defects produced during cosmological phase transitions has received a large amount of interest in recent years. Topologically stable kinks are ensured when the vacuum manifold of a spontaneously broken gauge theory is disconnected [1]. Let us consider, for simplicity, a model in which kinks are infinitely static domain walls in the zy -plane. That is we assume that the vacuum manifold consists of just two disconnected components.

The dynamics and gravitational properties of such defects are determined by their tension or surface energy density σ [2, 3]. Unless the symmetry breaking scale is very small, the surface density energy of the kink is extremely large and implies that cosmic domain walls would have an enormous impact on the homogeneity of the Universe. (Here and in the following for cosmic domain walls we shall mean walls of linear dimension H_0^{-1} , where H_0 is the Hubble constant). A stringent constraint on the wall tension σ for a cosmic \mathbb{Z}_2 -wall can be derived from the isotropy of the microwave background. If the interaction of walls with matter is negligible, then there will be a few walls stretching across the present horizon. They introduce a fluctuation in the temperature of the microwave background of order $\delta T/T \simeq 2\pi G\sigma H_0^{-1}$ [4], where G is the Newton's constant. Observations constrain $\delta T/T \lesssim 3 \times 10^{-5}$, and thus models predicting topologically stable cosmic walls with $\sigma \gtrsim 1\text{MeV}^3$ are ruled out.

In the following, we will see that the presence of a cosmic wall stretching our Universe modifies the electromagnetic properties of the free space. (This effect has been recently investigated in Ref. [5, 6, 7, 8] in the case of cosmic strings.) In particular, the gravitational field produced by a wall acts as a medium with time- and position-dependent permittivity. This means that the fine structure constant α , at fixed position, will be a time-dependent function. Because terrestrial experiments and observations constrain the time variation of α , we will be able to put a stringent limit on the energy density of a cosmic wall.

2. The Fine Structure Constant in the Spacetime of a Domain Wall

In this Section, we will see that the electric field generated by a charge particle in the spacetime of a domain wall is the same as in a flat spacetime but with a spacetime-dependent fine structure constant.

We start by writing the line element associated to the spacetime of a thin \mathbb{Z}_2 -wall [9]

$$ds^2 = e^{-4\pi G\sigma|x|}(dt^2 - dx^2) - e^{4\pi G\sigma(t-|x|)}(dy^2 + dz^2). \quad (1)$$

Given a general diagonal metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = g_{00}dt^2 - \gamma_{ij}dx^i dx^j \quad (2)$$

and the electromagnetic field strength tensor $F_{\mu\nu}$, the electric and magnetic fields in a curved spacetime are defined as [10]

$$E_i = F_{0i}, \quad B^i = -\frac{1}{2\sqrt{\gamma}}\epsilon^{ijk}F_{jk}, \quad (3)$$

where $\gamma = \det||\gamma_{ij}||$ is the determinant of the spatial metric and ϵ^{ijk} is the Levi-Civita symbol. (Here and in the following, Greek indices run from 0 to 3, while Latin indices run from 1 to 3.) The charge density of a particle of charge q at rest in the position $\mathbf{x} = \mathbf{x}_0$ is given by

$$\rho = (q/\sqrt{\gamma})\delta(\mathbf{x} - \mathbf{x}_0). \quad (4)$$

Introducing the fields

$$\mathbf{D} = \mathbf{E}/\sqrt{g_{00}}, \quad \mathbf{H} = \sqrt{g_{00}}\mathbf{B}, \quad (5)$$

the Maxwell's equations in three-dimensional notation read [10]

$$\text{div}\mathbf{B} = 0, \quad \text{curl}\mathbf{E} = -\frac{1}{\sqrt{\gamma}}\frac{\partial(\sqrt{\gamma}\mathbf{B})}{\partial t}, \quad (6)$$

$$\text{div}\mathbf{D} = 4\pi\rho, \quad \text{curl}\mathbf{H} = \frac{1}{\sqrt{\gamma}}\frac{\partial(\sqrt{\gamma}\mathbf{D})}{\partial t}, \quad (7)$$

where the divergence and curl differential operators are defined in curved spacetime by

$$\text{div}\mathbf{v} = \partial_i(\sqrt{\gamma}v^i)/\sqrt{\gamma} \quad (8)$$

and

$$(\text{curl}\mathbf{v})^i = \epsilon^{ijk}(\partial_j v_k - \partial_k v_j)/(2\sqrt{\gamma}), \quad (9)$$

respectively.

It is convenient to re-write the first equation of (7) as

$$\nabla \cdot (\varepsilon\mathbf{E}) = 4\pi q\delta(\mathbf{x} - \mathbf{x}_0), \quad (10)$$

where ∇ is the usual three-dimensional nabla operator in Euclidean space, and we have introduced the parameter $\varepsilon = \sqrt{\gamma}/\sqrt{g_{00}}$. The solution of Poisson equation (10) is the standard one:

$$\varepsilon \mathbf{E} = (q/4\pi r^3) \mathbf{r}, \quad (11)$$

where $\mathbf{r} = \mathbf{x} - \mathbf{x}_0$ and $r = |\mathbf{r}|$. Re-writing the above equation as

$$\mathbf{E} = \frac{q}{4\pi\varepsilon r^3} \mathbf{r}, \quad (12)$$

and taking into account the metric (1), we see that the gravitational field produced by a domain wall acts as a medium with permittivity ε given by ⁴

$$\varepsilon = e^{4\pi G\sigma(t-|x|)}. \quad (13)$$

In other words, the fine structure constant, defined in the free space as $\alpha_0 = e^2/4\pi$, becomes in the spacetime of a domain wall

$$\alpha = \frac{e^2}{4\pi\varepsilon}. \quad (14)$$

3. Discussion and Conclusions

From the above analysis it results that, if a cosmic wall were present within our Hubble horizon, then the fine structure constant would be time- and position-dependent. In particular, at fixed position, the time variation of α would be

$$\frac{\dot{\alpha}}{\alpha} = -4\pi G\sigma. \quad (15)$$

It is worthwhile noting that the “effective” variation of the fine structure constant, Eq. (15), is not in contradiction with the Einstein Equivalence Principle which implies that, locally in the spacetime, no variations of α can occur. ⁵ Indeed, what is measurable in our case are only differences of values

⁴In the case of Taub metric [11] (i.e. the most generic plane-symmetric metric) $ds^2 = e^{2u}(dt^2 - dx^2) - e^{2v}(dy^2 + dz^2)$, where u and v are functions of t and x , the permittivity induced by the gravitational field is $\varepsilon = e^{2v}$.

⁵It is well known [?] that the spacetime of a domain wall is locally flat everywhere except at $x = 0$. Therefore, one can perform a coordinate transformation such that the line element in Eq. (1) becomes that of a flat spacetime and, consequently, the Maxwell equations assume the “classical” form with $\varepsilon = 1$. Then, in agreement with the Einstein Equivalence Principle, no variation of the fine structure constant occurs locally in any point of the spacetime of a domain wall (excepting the points on the domain wall surface).

of α calculated at different spacetime points. Say in other words, only non-local variations of the fine structure constant are physical. Concerning this, it should be noted that all terrestrial experiments devoted to the detection of possible time variations of α measure, indirectly, values of α at different times. These terrestrial experiments set limits on the time variation of α [12]. Different experiments give different constraints which, however, are in the narrow range $|\dot{\alpha}/\alpha| < \text{few} \times 10^{-15} \text{ yr}^{-1}$ [13, 14, 15, 16]. This, in turns, gives a limit on the tension of a wall present in our Hubble volume, $\sigma \lesssim 1 \text{ MeV}^3$, which is of the same order of magnitude of that resulting from the isotropy of the microwave background.

The most stringent constraint on $\dot{\alpha}/\alpha$ comes from the natural reactor Oklo [17] and is $|\dot{\alpha}/\alpha| < \text{few} \times 10^{-17} \text{ yr}^{-1}$ [18]. This limit constrains the tension of a cosmic wall to be less than $\sigma \lesssim 10^{-2} \text{ MeV}^3$, and then represents the most severe limit on σ .

In conclusion, we have demonstrated that the gravitational field produced by a domain wall acts as a medium with spacetime-dependent permittivity and, consequently, the fine structure constant α is a time-dependent function at fixed position. Taking into account the most stringent constraint on the time-variation of α coming from the natural reactor Oklo, we derived an upper limit for the tension of a cosmic domain wall. This represents the strongest upper limit on the energy density of a cosmic wall stretching our Universe to date.

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